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Anton Bovier: Statistical mechanics of disordered systems. A mathematical perspective

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Disordered spin systems are an extremely active field of research, mostly because of the rich exchanges of ideas and inspiration among theoretical physics, mathematical physics and probability theory. The book "Statistical Mechanics of Disordered Systems. A Mathematical Perspective" by Anton Bovier is an excellent example of this. The author, who may be equally well considered as a mathematical physicist or as a probabilist (and who started his career as a theoretical physicist) has given fundamental and acknowledged contributions to this field. This book is a way to make some recent developments available to a wider public than just specialists, and to put them as far as possible in a common perspective.

Part I, "Statistical mechanics", is aimed mainly at making the book self-contained, and the material is rather standard (Gibbs formalism, Peierls' argument, etc.). An exception is Chap. 5, "Cluster Expansions", which treats a topic not often covered by reviews. I found this introduction to high- and low-temperature expansions rather enjoyable. Even independently of the rest of the book, Part I can be quite useful for a student who wishes to get acquainted with the language and formalism of mathematical statistical mechanics.

With Part II, "Disordered Systems: lattice models", one comes to the heart of the subject: random spin models (here, in finite dimension). Chapter 6 is a valuable presentation of the concept of *metastate*, introduced originally by M. Aizenman and J. Wehr, and explored in depth by C. Newman and D. Stein. The metastate, which looks quite abstract at first sight, is a central object for disordered systems (and appears in later sections of the book, when the Random Energy Model (REM) and the Hopfield model are studied). Chapter 7 is dedicated to the Random Field Ising Model. In particular the author presents in a detailed and clear way the beautiful argument by M. Aizenman and J. Wehr, which shows that in two dimensions an arbitrarily small amount of random magnetic field eliminates the spontaneous magnetization of the standard Ising model. If dimension is larger than or equal to three and the random field is weak, on the contrary, the spontaneous magnetization is different from zero at low temperature. These are the contents of a famous work by J. Bricmont and A. Kupiainen,

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based on a rigorous renormalization group (RG) scheme. This RG procedure is exposed in Sect. 7.3 in a pedagogical effort to present its main ideas, leaving aside purely technical estimates.

Part III, "Disordered systems: mean-field models" is mainly dedicated to mean field spin glasses. Chapters 9 and 10 treat the REM and GREM models introduced by Derrida, and include less standard topics like the "Continuous random energy model" studied in particular by A. Bovier and I. Kurkova, and the Bolthausen-Sznitman coalescent process. Poisson-Dirichlet point processes as description of the infinite-volume Gibbs distribution play a fundamental role here. REM and GREM, beyond being mean field models, have the special feature that they have a built-in hierarchical structure. This is not the case for the Sherrington-Kirkpatrick (SK) model (Chap. 11), where the hierarchical structure (ultrametricity) emerges spontaneously in the thermodynamic limit. While the proof by M. Talagrand that the free energy of the SK model coincides with G. Parisi's "Replica Symmetry Breaking solution" is not given here, one can find the proof of F. Guerra's result which states that the Parisi functional provides a free energy upper bound (the proof is presented by A. Bovier in the beautiful cavity framework of Aizenman-Sims-Starr). Chapter 12 is dedicated to the Hopfield model of associative memory, which mathematically can be seen as a generalization of SK, where couplings are no more independent. Results on the Hopfield model (a subject on which A. Bovier and collaborators have given fundamental mathematical contributions) are presented, starting from the case where the number M of memorized patterns is finite (in which case more or less standard large deviation techniques can be used) up to the case where M grows linearly with the number of neurons. Finally, the last chapter focuses on the "Number Partitioning Problem" (NPP), seen as a spin glass problem. This can be the occasion for the reader to make a bridge between spin glasses and *combinatorial optimization problems*, a field which is presently in a phase of tremendous activity both in computer science and in statistical mechanics, and where the NPP represents a particularly simple and instructive example.

Even if much of the focus of "Statistical Mechanics of Disordered Systems" is on spin glasses, it treats the subject in a spirit and with a style which is quite different from that of the recent and well-known "Spin glasses: a Challenge for Mathematicians" by M. Talagrand; in this sense the two books should be seen as complementary rather than as rivals. Overall, the value and usefulness of the book by A. Bovier for anyone interested in the rigorous theory of disordered (spin) systems can be hardly overestimated.